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1989 J. Phys.: Condens. Matter 1 5811

(http://iopscience.iop.org/0953-8984/1/33/027)

J. Phys.: Condens. Matter **1** (1989) 5811–5815. Printed in the UK

LETTER TO THE EDITOR

Coulomb oscillations of the conductance in a laterally confined heterostructure

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Received 13 June 1989

Abstract. We predict a new type of conductance oscillations in a GaAs heterostructure with a proposed strip-like gate with a hole. Such a gate defines a conducting dot (under the hole) in an otherwise depleted region of a two-dimensional electron gas. The oscillations are caused by discrete changes of the charge of the dot as the gate voltage is varied. We show that for low temperatures, $T \leqslant e^2/2C$, the conductance as a function of gate voltage consists of a series of peaks with a period $\Delta V_G \sim e/C$ (here C is the capacitance of a dot). A distinct feature of the predicted oscillations (in comparison with the quantum ballistic interference phenomena) is the weak dependence on magnetic field and carrier scattering.

The considerable present attention to ballistic transport in small systems is connected with a recently developed method for electrostatic confinement of the two-dimensional electron gas (2DEG) in GaAs-based heterostructures. This confinement is provided by a gate, the size and shape of which determines the geometry of the barriers for the 2DEG. Using appropriate gate structures a type of tunable microconstriction can be created. These have enabled one to discover fundamental conductance steps caused by quantisation of the transverse motion of an electron in the microconstriction [1, 2]. Recently [3] the possibility of creating disconnected structures was demonstrated. In these structures electrons are localised in an area inside a ring-shaped potential barrier (figure 1). The presence of this barrier blocks the charge relaxation processes and leads to an increased role for Coulomb correlation phenomena in the electron transport process. The sub-micron size of the localisation area, R^2, makes the discrete nature of the electronic charge important. We show that the discreteness results in a new type of conductance oscillation. These oscillations are related not to the spatial quantisation of electron states but to the Coulomb energy (of order $e^2/\varepsilon R$). In contrast to the interference phenomena, they are not destroyed by electron scattering and are much less sensitive to a magnetic field. Coulomb phenomena should be taken into account even in a clean sample with a size, R, that is larger than the effective Bohr radius, a_B, because the period of oscillations is determined mainly by the charging energy (instead of the spatial quantisation).

A negative gate bias V_G, figure 1(a), leads to the creation of a tunnel structure with a conducting dot separated from the 2DEG in leads, figure 1(b). Simultaneously a finite

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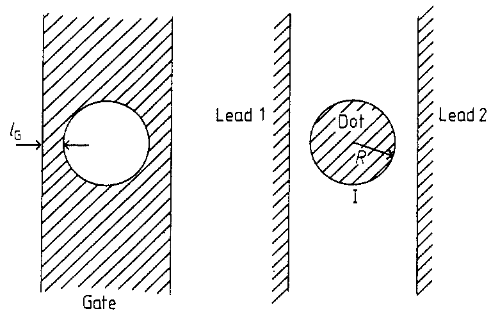

Figure 1. (*a*) The gate geometry and (*b*) the geometry of a dot surrounded by the depletion area.

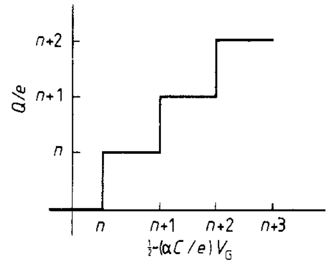

Figure 2. The dependence of an excess charge q on the dot as a function of the gate voltage V_G under $T = 0$; n is an integer number.

V_G lead to a potential difference between the dot, I, and the leads, 1, 2, figure 1(*b*). This difference forces some of the electrons to leave the dot and hence the dot acquires an excess charge. However, the only discrete changes of the charge are possible, i.e. the dependence of the excess charge, Q, on the voltage V_G is of a step-like type (figure 2). At the points of discontinuity of the function $Q(V_G)$, a charge degeneracy occurs. The change of the number of electrons in a dot by one does not alter the Coulomb energy [4, 5]. This degeneracy opens up a channel for unactivated transport of electrons through a dot. For other values of V_G changes of the dot charge and electron transport occur only as activated processes†. The result is a system of peaks in the conductance $G(V_G)$ corresponding to steps in the $Q(V_G)$ function.

The Coulomb energy of a dot can be written as

$$E_n(\varphi) = Q^2/2C + Q\varphi. \tag{1}$$

Here $Q = ne$ is the discrete charge of a dot, φ is the potential of the dot produced by the charges of surrounding conductors (the gate and the leads, figure 1), C^{-1} is a diagonal element (associated with the dot) of the inverse capacitance matrix $\boldsymbol{\alpha}_{kl}$ [6]. We shall take the potential of both leads to be zero in the absence of a current. For this gauge the voltage φ is proportional to the gate bias V_G (all potentials are measured from the zero potential of the leads). The simplest relation between φ and V_G corresponds to the limit of large leads

$$\varphi = \alpha V_G \qquad \alpha = \alpha_{GI}/\alpha_{GG} < 1 \tag{2}$$

($\alpha_{1I} \sim \alpha_{2I} \rightarrow 0$; 1, 2, G, I are the indexes of leads, gate and dot respectively). The dot is in electrical contact with the leads due to the tunnel coupling. Hence, the number of electrons in the dot is not fixed. In the absence of a current this number is given by an equilibrium distribution function

$$W_0(n) = \exp(-\beta E_n(\varphi)) \Big/ \sum_n \exp(-\beta E_n(\varphi)) \qquad \beta = 1/T. \tag{3}$$

It follows from equations (1)–(3), that the average value of the dot charge $q = \langle Q \rangle$ depends on the gate bias V_G:

$$q(V_G) = \sum_n enW(n). \tag{4}$$

The properties of the function $q(V_G)$ depend substantially on the parameter γ:

$$\gamma = e^2/2CT. \tag{5}$$

At high temperatures ($\gamma \ll 1$) the sum in (4) can be replaced by an integral, and $q(V_G) =$

† We neglect here the possibility of quantum tunnelling via a virtual state with an extra carrier in the dot.

$-\alpha CV_G + e/(\pi\gamma)^{1/2}$ becomes a linear function of V_G. In the opposite limit of low temperatures, $\gamma \gg 1$, thermal fluctuations are suppressed by the Coulomb energy. For $\alpha CV_G \neq (n + \frac{1}{2})e$ there exists only a *single* charging state with a minimum energy $E_n(\varphi)$. This state is now the only one of importance in the sum over states n in (4). Under the condition $\alpha CV_G = (n + \frac{1}{2})e$ the degeneracy mentioned above occurs and *two* different charging states give the same minimum energy. For low temperatures, the function $q(V_G)$ takes the form:

$$q(V_G) = e[x] + \tfrac{1}{2}e \exp \gamma(\{x\} - \tfrac{1}{2})/\cosh \gamma(\{x\} - \tfrac{1}{2}) \qquad x = \alpha CV_G/e. \tag{6}$$

In this equation $[x]$, $\{x\}$ are the integer and fractional parts of x, respectively. It is obvious from (6) that the system periodically goes into the degenerate state as the value of q changes. Simultaneously, the conductance of the system becomes unactivated.

The source–drain voltage applied between the terminals 1, 2 causes a current flow through the dot. Under this condition, the charge distribution differs from that in equilibrium and the function $W(n)$ should be determined from the stationary kinetic equation. This equation was derived in [4] using a tunnel Hamiltonian formalism that included transfer of an electron between the dot and both leads. This equation can be presented as follows:

$$\begin{aligned} &F_{n+1} - F_n = 0 \\ &F_n = \{g_{1I} f[E_n(\varphi) - E_{n-1}(\varphi) - eV_1] + g_{2I} f[E_n(\varphi) - E_{n-1}(\varphi) - eV_2]\}\, W(n) \\ &\qquad - \{g_{1I} f[E_{n-1}(\varphi) - E_n(\varphi) + eV_1] \\ &\qquad + g_{2I} f[E_{n-1}(\varphi) - E_n(\varphi) + eV_2]\} W(n-1) \end{aligned} \tag{7}$$

where g_{1I}, g_{2I} are tunnel conductances of the barriers separating the dot from the leads 1 and 2; $f(x)$ is a standard function for tunnelling problems that correspond to Fermi's Golden Rule:

$$f(x) = \int \mathrm{d}E\, n_F(E + x)[1 - n_F(E)] = x/[1 - \exp(-\beta x)]. \tag{8}$$

In a stationary state, the incoming and outgoing currents through the dot do coincide and to calculate the current I it is sufficient to consider one element of the chain, e.g. the link $I \to 2$:

$$\begin{aligned} I = e^{-1} g_{2I} \sum_n \{&f[E_{n-1}(\varphi) - E_n(\varphi) + eV_2] W(n-1) \\ &- f[E_n(\varphi) - E_{n-1}(\varphi) - eV_2] W(n)\}. \end{aligned} \tag{9}$$

We are interested in a linear conductance problem. That is why the functions f in (7) should be linearised with respect to V_1 and V_2. Then a linear correction to the equilibrium function (3) can be determined from (7) and inserted in (9). These calculations bring a result for the conductance

$$G(V_G, T) = \frac{g_{1I} g_{2I}}{g_{1I} + g_{2I}} \frac{1}{T} \sum_n f[E_n(\varphi) - E_{n-1}(\varphi)] W_0(n). \tag{10}$$

To be consistent with the linear expansion mentioned, we have to neglect the influence of the potentials V_1, V_2 on the value φ. Hence σ is determined by (2). For high temperatures ($\gamma \ll 1$), (10) leads to the usual formula for two resistances in series: $G(V_G, T \to \infty) \equiv G(V_G, \infty) = (g_{1I} g_{2I})/(g_{1I} + g_{2I})$. At low temperatures there is only one main contribution in the sum (10). If $\alpha CV_G \neq (n + \frac{1}{2})e$, then the conductance is exponentially small, $G \simeq \exp(-\gamma)$, which corresponds to thermal activation over the Coulomb barrier. This barrier vanishes at $\alpha CV_G = (n + \frac{1}{2})e$. Finally, for low temperatures ($\gamma \gg 1$) one finds:

$$G(V_G, T)/G(V_G, \infty) = \gamma(\{x\} - \tfrac{1}{2})/\sinh 2\gamma(\{x\} - \tfrac{1}{2}) \qquad x = \alpha CV_G/e. \tag{11}$$

Here $\{x\}$ is again the fractional part of x. The function $G(V_G)$ consists of a system of peaks at $V_G \equiv V_n = (e/\alpha C)(n + \frac{1}{2})$. The width of the peaks is proportional to the temperature. Fully analogous oscillations with a period of order e/C occur also in the dependence of conductance on source–drain voltage. This gives rise to a step-like behaviour of the current–voltage characteristics at the low temperatures, as was predicted in [4].

In order to discuss the possibility to observe the Coulomb oscillations (11) in laterally-confined heterostructures, we have to mention the double role of the variations in V_G. Besides periodically suppressing the Coulomb barrier (as indicated by (11)), the gate bias also lowers the transparency of the barriers. Hence the oscillatory dependence of the conductance, $G(V_G)$, manifests itself as a series of peaks with exponentially decaying amplitudes (but the width of the peaks remains constant and is of order T). It is possible to estimate the decrease of the transparency when an oscillation period $\Delta V_G = e/\alpha C$ is added to a value of V_G. Introduce the critical value V_G^* of the gate voltage that corresponds to the creation of depleted areas between the dot and the leads. For $|V_G| > |V_G^*|$ tunnel barriers for the electrons are formed. The height of these barriers is of order $|V_G - V_G^*|$ their widths are determined by the gate width†, l_G. Hence, the argument of the dominant exponential function in the expression for the tunnelling probability can be estimated as

$$S \sim l_G(m|V_G - V_G^*|)^{1/2} \tag{12}$$

(m is an effective electron mass). The ratio G_n/G_{n-1} of amplitudes of the sequential peaks can be determined from (12) by using $|V_G - V_G^*| = n\Delta V_G$:

$$\ln(G_n/G_{n-1}) \sim (l_G/a_B)(\alpha\varepsilon a_B/C)^{1/2}[n^{1/2} - (n-1)^{1/2}]. \tag{13}$$

Let us estimate the value of the period ΔV_G for a submicron structure with a dot of typical size $2R \simeq 0.3\ \mu$m. Approximating the shape of the dot by a circular disc, and using $\alpha = 1$ one finds $\Delta V_G \simeq 1$ mV. For a minimum value $l_G \sim 800$ Å, equation (13) gives $\ln(G_2/G_1) = 1.6$.

The rapid decay of the amplitudes (13) demonstrates that although the Coulomb oscillations occur in a regime of tunnelling conductance, they require the deviation of the gate voltage V_G from the critical value V_G^* to be small. On the other hand, for *ballistic* transport in the range of under-critical gate voltages, $0 < |V_G^*| - |V_G| \ll |V_G^*|$, there is a possibility of oscillations of a different nature [3, 7]. These are caused by resonant reflection of the electrons passing over the barrier. However, these oscillations should be destroyed when a relatively weak magnetic field is applied to the system. This feature determines the difference between these and the Coulomb oscillations. In this context, it is interesting to mention the experiment [3] with a geometry similar to that in figure 1. Two groups of oscillations superimposed on the monotonic increase of the resistance R as a function of V_G were observed [3]. The first group corresponds to a region with a relatively slow increase of R and the oscillations were easily suppressed by a magnetic field. For the second group (that corresponds to a rapid increase in R (V_G) with V_G) no sensitivity to a magnetic field was mentioned. A possible explanation of such a behaviour can be related to the Coulomb oscillations discussed in this Letter.

† We assume that l_G exceeds the distance between the gate plane and the plane of 2DEG. The latter is usually of order of 800 Å.

In conclusion, we have studied the dependence of the conductance, G, on the gate voltage, V_G, for a disconnected 2D electron system produced by a strip-like gate with a hole in it (figure 1). This dependence reveals an oscillatory pattern in a region where V_G slightly exceeds a critical value necessary for the formation of a quantum dot. We argue that the period of this oscillatory pattern is determined by charging phenomena rather than by spatial quantisation. A weak dependence on the magnetic field should be a characteristic feature of these oscillations.

One of the authors (LG) would like to acknowledge the hospitality of the Institute of Theoretical Physics, Chalmers University of Technology. He is especially indebted to Dr Mats Jonson for his help.

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